AN ESSAY

ON THE

PRESENT STATE OF ASTRONOMICAL CERTAINTY,

WITH REGARD TO THE QUANTITY OF THE EARTH'S MAGNITUDE, THE DISTANCE OF THAT PLANET FROM THE SUN, AND THE ABSOLUTE LIMIT OF THE SMALLEST POSSIBLE INTERVAL FROM THE SUN TO ANY ONE OF THE FIXED STARS.

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To ascertain the dimensions and distances from each other, of the various bodies that compose our solar system, is a problem which, we find, has exercised the ingenuity of Astronomers, from the period of the earliest records we have, of the application of trigonometrical calculations to the improvement of their useful and sublime science. To determine the various questions involved in this enquiry, an actual knowledge of the distance of the earth from the sun, is first required; an element of great importance in this, and, indeed, in many other branches of astronomical calculation. As the works of Ptolemy make no mention of any Chaldean or Indian attempts at the solution of this problem, it is probable that none such existed in his time, as there can be no doubt, that

that were there any such upon record, they would have been deposited in the celebrated library of Alexandria, and would, of course, have been noticed in his great work. This discovery, however, was very early attempted by two eminent Astronomers of the Greek School, who, each of them, suggested a different method of determining the distance of the earth from the sun, both highly ingenious and strictly geometrical, but practically insufficient, from their requiring observations of such minuteness and accuracy, that not even the improved state of modern optical instruments can enable us to attain to the precision they demand. A necessary step to the most direct and certain solution, of this nice question, is to determine the angular quantity of the difference of the sun's apparent place in the Heavens, as seen at the same instant, from the centre and the surface of the earth, usually called the angle of the sun's parallax; for it is well known to Astronomers, that this quantity is equal to the angle under which the earth's semidiameter is seen from the sun; and that, were *this angle*, and the measure of the mean semidiameter of the earth, also given; from these data, by the help of plain trigonometry, the distance of the sun, and, of course, that of every primitive planet belonging to our system, may be readily determined. But this angle is too small for direct observation; a very close approximation to the discovery of it, has been made by various acute and well-conceived, though indirect methods. The parallactic angle of the planet Mars, the theory of gravitation, and, above all, the two recent observations of the transits of the planet Venus over the sun's disk,

disk, have been made subservient to this purpose, and, indeed, have reduced any remaining uncertainty about the quantity of this angle, within very narrow limits; so that, in fact, it may be considered with almost actual certainty, that the angle of the sun's horizontal parallax, at the mean distance from the earth, lies between the limits of $8''{,}65$, and $8''{,}75$.

To enable persons, who are not in the habit of making astronomical calculations, to judge of the accuracy to which the distance of the earth from the sun has been hitherto determined, I have arranged the following short table, which will shew at one view, within what bounds, our present knowledge on this subject is comprised, and to what errors we are still liable. It may, perhaps, also, from a view of the uncertainty that still hangs over this subject, suggest to those who are qualified for such undertakings, to obviate the remaining imperfections, and thus attain to still higher degrees of precision, in ascertaining this important element of astronomical knowledge.

A TABLE

A Table of the various possible distances of the Earth from the Sun, according to the present limits of the best observations that have been made of the horizontal parallaxes of the Sun, and the mean semidiameters of the Earth.

HORIZONTAL PARALLAXES OF THE SUN.

Semidiameters of the Earth in British miles. Three thousand, nine hundred, and	Miles.	8″. 6.	8″. 7.	8″. 8.
	45	94.618.000.	93.531.000.	92.467.000.
	46	642.	554.	491.
	47	666.	578.	514.
	48	690.	602.	537.
	49	714.	625.	561.
	50	738.	649.	585.
	51	762.	672.	608.
	52	786.	696.	632.
	53	810.	720.	653.
	54	834.	743.	677.
	55	858.	767.	700.
	56	882.	791.	724.
	57	906.	815.	747.
	58	930.	838.	771.
	59	954.	862.	795.
	60	978.	886.	819.
	61	95.001.000.	909.	843.
	62	025.	932.	866.
	63	049.	955.	889.
	64	073.	978.	914.
	65	097.	94.002.000.	936.

As

As the assumed quantities (from whence the distances, as given in this table, are calculated) are supposed to extend to any possible degree of uncertainty in either element, it may not be improper to mention on what grounds these limits are severally conceived to be so ascertained. The mean semidiameters of the earth, are deduced from a mean of several very excellent and careful mensurations of degrees of the terrestrial meridians, such as those of Picard and Norwood, in France and England; of Snellius, in Holland; of De Ulloa, De la Condamine, &c. in S. America; Maupertuis and his associates, in Lapland; and the more recent ones of Cassini, General Roy, Lalande, and Lambton. To give a satisfactory view of the actual progress already made, towards ascertaining this element of our calculation, I have subjoined a table of the mean semidiameters of the earth, obtained either by the actual mensuration of degrees near the probable situation of the mean radius of the earth, or else taken from extreme measures reduced to means on the most approved hypothesis of the figure of the earth, and the ratios of its radii, with the names of the mensurators and calculators; from the result of which it will appear, that the accuracy of the determination of the quantity of the earth's semidiameter, lies within the probable limits assigned to it in my table of the possible distances of the earth from the sun.

The deductions in the following table are compared with a mean semidiameter of the earth, deduced from the table of degrees of latitude in French toises, in Vince's Astronomy,

Vol. II. p. 110, calculated upon the hypothesis, that the æquatorial diameter of the earth exceeds the polar, by $\frac{1}{310}$ of the whole, and that 825,7095 French toises equal a British mile. The mean semidiameter from the table is 3956,95 British miles.

A Table exhibiting the results of different measures on the Earth's Surface, &c. &c.

Names of the Measurers.	*Place of Measurement.*	*Time of do.*	*Authorities.*
Maupertuis & Company,	Lapland.	1736,7.	Robinson's Nagivation.
D'Ulloa and Condamine,	South America.	1736,44.	Vince's Astronomy.
Cassini,	France.	1750.	Robinson's Navigation.
De la Caille,	France.	1750.	Vince's Astronomy.
Boscovitch,	Italy.	1755.	Robinson and Vince.
De la Caille,	Cape of Good Hope.	1752.	Robinson's Navigation.
Mason,	North America.	1766.	Vince's Astronomy.
Dixon,	North America.	1766.	Vince's Astronomy.
Liesganig,	Austria & Hungary.	1784,85.	Phil. Trans. 1791.
General Roy,	England.	1787,88.	Phil. Trans. 1790.
Brigade Major Lambton,	Madras.	1803.	

Continuation

Continuation of the Table, &c.

Names of the Measurers.	*Lat. of middle point.*	*Measured deg. in toises.*	*Deg. from Tab. in toises.*	*Mean Semid British miles.*
Maupertuis & Company,	66°. 20′. N.	57438.	57222.	3971,35.
Ulloa and Condamine,	0 0 0	57422. 56757.	56747.	3957,7.
Cassini and De la Caille,	49°. 20′. N.	57074. 57069.	57070.	3957,08.
De la Caille and Cassini,	45°. 0.	57050. 57028.	57026.	3965,3.
De la Caille,	33°. 18′. S.	57037.	56919.	3954,72.
Boscovitch,	43. N.	56972. 56979.	57007.	3954,95.
Mason and Dixon,	39′. 12′. N.	56888.	56969.	3951,4.
Liesganig,	48°. 43′. N.	57086.	57068.	3958,2.
General Roy,	50°. 9′. 30. N.	57075.	57078.	3956,8.
Brigade Major Lambton,	12°. 32. N.	56762,8.	56770.	3956,5.

The mean semidiameter of the earth from a mean of *all* is 3958,69 British miles, and the mean semidiameter, from a measurement executed under the direction of the French National Institute, which extended from Dunkirk to Barcelona, and was made between the years 1791 and 1798, as reduced by Lalande, is equal to 3958 British miles.*

With regard to the quantity of the other element in this calculation, namely, the Sun's horizontal parallax, it is to be observed, that the determination of this quantity is most correctly deduced, from a comparison of the *observed* and *calculated* effects of the solar parallax, upon the several phænomena of the transits of the planet Venus over the Sun, especially on the apparent times of the internal and external contacts of the limbs of the planet with those of the Sun. The observations of the external contacts of the Sun's and Venus's limbs, were very carefully and judiciously made at the observatories

* Mr. Dalby, in an elaborate paper published in the 81st Vol. Philosophical Transactions, read May 19, 1791, gives his determination of the longitude of Dunkirk and Paris, from the triangular measurements made in the years 1787 and 88, by the late General Roy, on the supposition of the earth's being an elipsoid, whose magnitude is determined by adhering *nearly* to the measured arc of the meridian between Greenwich and Paris, obtained by the aforesaid operations. On which hypothesis it will appear, that the measured degrees of the meridian in middle latitudes, agree, very nearly indeed, to the assumed elepsoid, whose axes are to each other in the ratio assigned by Sir Isaac Newton, viz. 229 to 230, and gives a mean semidiameter of the earth, of 3956,55 English miles; a quantity, most probably, very near to precision.

servatories of Philadelphia and Norriton, in North America, and are stiled by our learned astronomer, Dr. Maskelyne, " complete and excellent observations;" and compared with those made by skilful persons at the settled observatories in Europe, under favorable circumstances of both weather and latitude; as also with others made at the Cape of Good Hope, the East Indies, and the Island of Otaheite, in the South Seas; in all of which places observations of *either*, or both, the transits of the years 1761 and 1769, were obtained, gives results which, probably, are the best and most accurate which will be got for centuries to come, towards the elucidation of this nice and interesting problem. The following summary of these rusults, will serve to give a clear and concise view of the degree of accurary we may count upon, in regard to this element.

Mean

Mean horizontal Parallaxes of the Sun, as deduced from the best observations of the late transits of the planet Venus over the Sun's disk.

	"
From the American observations of 1761, - - - - - - - -	8,6500
From the American observations of 1769, - - - - - - - -	8,6045
From a Mean of Mr. Short and Dr. Hornsby's best observations,	8,8500
By professor Euler's deductions, - - - - - - - - - - - -	8,6800
By those of Mr. Pingré, - - - - - - - - - - - - - - -	8,8000
Mr. Lexel, - - - - - - - - - - - - - - -	8,6300
Mr. Sejour, - - - - - - - - - - - - - - -	8,8100
Mr. Lalande, - - - - - - - - - - - - - - -	8,6000
A mean of these eight deductions gives - - - - - - - -	8,703.

For the sun's mean horizontal parallax, which is very near the single result obtained by Dr. Maskelyne, from a comparison of the observations made at Otaheite, and Wardhus, viz. 8",72.

From

From a view of the table of the possible distances of the earth from the sun, it will appear, that with a given parallax, each difference of an English mile in the semidiameter of the earth, will occasion a difference of about 24,000 English miles in the distance, and that the semidiameter remaining the same, the difference of a tenth of a second in the horizontal parallax, alters the distance of the earth from the sun above *one million of miles*: hence, we may conclude, the immense distance of the fixed stars from our planet, as no one of them has yet been discovered to be affected by any sensible parallax; notwithstanding, that to enable us to detect any such apparent change of place in a fixed star, as is called *a parallax*, we can, by the earth's describing in one year an orbit round the sun of nearly 190 millions of miles in diameter, in the course of six months, observe a fixed star's distance from the zenith, at a station that is no less than 190 millions of miles distant from that at which we observed its zenith distance six months before. But, as no sensible change of a star's zenith distance is observable on this account, it is evident, that the whole diameter of the great orb of the earth's annual motion round the sun, subtends no discernible angle, as seen from the *nearest fixed star*. If a fixed star had a sensible parallax of even one second, still, the distance of that star from the sun, would be above 400,000 times the distance of the earth from the sun. And, if a fixed star had a sensible diameter of one second, and also a sensible parallax of one second, then, its actual dimensions would equal the radius of the orb of the sun's distance from the earth.

earth. Thus, by the doctrine of parallaxes, when they are *sensible*, we find the absolute distances of the heavenly bodies; and when they are even *insensible*, we obtain from them, a sort of negative information, which determines, that bodies so circumstanced, must, at least, in their remoteness, exceed certain limits, which we can easily assign. For thus much is *certain*, that as no one of the fixed stars has any sensible parallax, the distance of the nearest of them must exceed twice the distance of the earth from the sun, multiplied by 206,264, a distance which will be more distinctly conceived by actually so multiplying any of the distances given in the table, than by a mere inspection of the product. The product of such a multiplication, supposing the double distance of the earth from the sun to be in round numbers, 190 millions of miles, will consist of fourteen figures. This multiplier* will give nearly the same number of miles for the nearest of the fixed stars, as may be deduced from the following correct analogy, supposing the same diameter of the annual orb, as above stated, and a parallax in the fixed star of one second.

As the tangent of 1″ : radius : : 190 millions of miles : distance = to 39191000000000 British miles!

* This multiplier is the seconds in an arch equal to Radius.